A Three-Stage Meta-Heuristic-Based Framework for Voltage Estimation in Low-Voltage Distribution Grids

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Abstract

This paper proposes a three-stage optimization-based methodology for estimating future operating conditions in low-voltage distribution networks with limited observability. The framework formulates short-term grid state estimation as a network optimization problem under forecasting and structural constraints. The proposed workflow consists of: (i) node-level forecasting of active and reactive power demands over a 24-hour horizon, (ii) phase-level disaggregation of aggregated nodal forecasts, and (iii) voltage state estimation solved via a hybrid metaheuristic optimization scheme. The hybrid solver integrates greedy heuristics for initialization, particle swarm optimization for global exploration, local search for solution refinement, and simulated annealing for escape from local optima. The methodology is designed to operate within a digital twin environment and explicitly targets enhanced grid observability and computational efficiency in large-scale low-voltage networks. By combining intelligent forecasting with hybrid network optimization, the proposed approach improves voltage estimation accuracy while maintaining tractable computational complexity. Simulation results show that the method achieves a root-mean-square error (RMSE) of 1.156 V, mean absolute error (MAE) of 0.908 V, standard deviation of 1.060 V, and a maximum voltage error of 4.124 V, demonstrating the effectiveness and reliability of the hybrid metaheuristic optimization framework.

Keywords

Network optimization, hybrid metaheuristics, Distribution System State Estimation

1 Introduction

The concept of digital twins in power systems refers to a digital representation of physical grid assets that dynamically mirrors their real-world behaviour through continuous data assimilation and computational models. From a network optimization perspective, a digital twin can be viewed as a decision-support layer that enables real-time inference, prediction, and control of complex networked systems. In low-voltage (LV) distribution grids, the rapid proliferation of renewable energy sources (RESs), electric vehicles (EVs), and distributed generation (DG) has significantly increased network heterogeneity [1], uncertainty, and operational complexity, thereby amplifying the need for enhanced observability and scalable computational tools [2]. Digital twins address these challenges by synthesizing limited and heterogeneous measurements into coherent network-level representations, enabling effective monitoring, optimization, and control while preserving reliability and efficiency.

Digital twin lies the problem of state estimation (SE) At the core of any grid which aims to infer nodal voltages and network states from incomplete and noisy measurements. From a network-optimization viewpoint, SE constitutes a large-scale, constrained inference problem defined over graph-structured systems, where computational tractability, robustness to uncertainty, and scalability are of paramount importance. Consequently, the development of advanced SE methodologies has evolved in close connection with optimization algorithms for networked systems, forming a foundational component of modern LV digital-twin architectures.

Initial efforts of SE focused on deterministic formulations and linear Kalman filtering to estimate voltage magnitudes and phase angles in transmission networks, constrained by the limited computational capabilities of early digital computers. Then attention shifted toward dynamic state estimation (DSE) to capture time-varying loads, switching actions, and network reconfigurations. Extended Kalman filters and related recursive algorithms were developed to address nonlinearity and bad-data detection, enhancing robustness in real-time applications [3],[4],[5],[6].

Hybrid optimization frameworks that combine physics-based network models with advanced algorithmic strategies have been adopted. These include decomposition-based methods for large-scale Distribution System State Estimation (DSSE) problems using Lagrangian or related partitioning techniques [7], real-time recursive estimation embedded within digital-twin environments, and data-driven schemes leveraging deep learning and graph neural network architectures. For example, hybrid graph-neural-network-based methods have been proposed to approximate nonlinear state mappings while preserving physical consistency [8] and to handle topology changes and limited observability in power networks [9, 10]. Deep ensemble learning approaches further combine statistical models with real-time estimation frameworks to improve accuracy and robustness under noisy measurements [11]. These methods represented an early integration of optimization and system dynamics, significantly improving estimation accuracy in evolving network conditions.

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Recent studies have demonstrated the effectiveness of optimization-based techniques for estimating critical parameters and states in distribution systems [12], [13], [14]. These approaches formulate estimation tasks as nonlinear optimization problems, enabling the systematic handling of measurement noise, modeling uncertainties, and limited observability. Metaheuristic and hybrid optimization methods have been shown to further enhance estimation accuracy and convergence robustness in distribution systems. In [12], the Whale Optimization Algorithm is employed to estimate transmission line parameters from voltage and current measurements, demonstrating the capability of bio-inspired metaheuristics to handle nonlinear and noisy estimation problems. Building on this direction, [13] proposes a nested Particle Swarm Optimization (PSO) framework with adaptive hyperparameters to improve convergence behavior and solution quality in distribution system state estimation. In [14], PSO is integrated with an adaptive neuro-fuzzy inference system to optimally tune model parameters for voltage stability margin estimation, highlighting the benefits of coupling learning-based models with global optimization. Similarly, a hybrid approach is introduced in which a shallow neural network provides informed initialization for a Gauss–Newton algorithm, significantly improving convergence reliability and computational efficiency. These studies highlight a growing trend toward hybrid and metaheuristic-driven estimation frameworks, where optimization algorithms play a central role in overcoming nonlinearity, nonconvexity, and data limitations inherent to modern distribution systems.

While existing approaches improve estimation accuracy under low-observability conditions, they often face challenges in computational scalability, convergence, and real-time deployment in large low voltage networks. Motivated by these limitations, this paper proposes a hybrid optimization framework that integrates forecasting, phase-level disaggregation, and voltage state estimation within a digital twin environment. The main contributions are:

1. (1) a three-stage workflow combining forecasting, phase disaggregation, and network optimization;
2. (2) a hybrid metaheuristic algorithm that integrates greedy heuristics, particle swarm optimization, simulated annealing, and local search to balance estimation accuracy and computational efficiency;
3. (3) a design tailored for practical deployment in real-world low-voltage digital-twin applications.

2 Preliminaries

In this section preliminaries for the problem and solution method are discussed.

2.1 System Overview

Figure 1 illustrates the single-line representation of the studied three-phase unbalanced distribution grid and the spatial placement of measurement devices across the network. The test system consists of 50 nodes that are interconnected through distribution lines and a distribution transformer, forming the underlying network structure used for DSSE analysis. The unbalanced nature of the grid is explicitly considered by modeling each phase independently, reflecting realistic operating conditions in low-voltage distribution networks. The teal marker denotes the meter installed at the secondary side of the distribution transformer, which provides three-phase voltage measurements. The network is supplied through a 400 kVA transformer, whose electrical characteristics are summarized in Table 1. Nodes equipped with

Table 1: Transformer parameters

V (kV)	S (kVA)	Conn	No-load (kW)	Load (kW)	I_0 (%)	S_U (kV)
10.4/0.4	400	DYN5	0.598	3.87	0.14	0.4

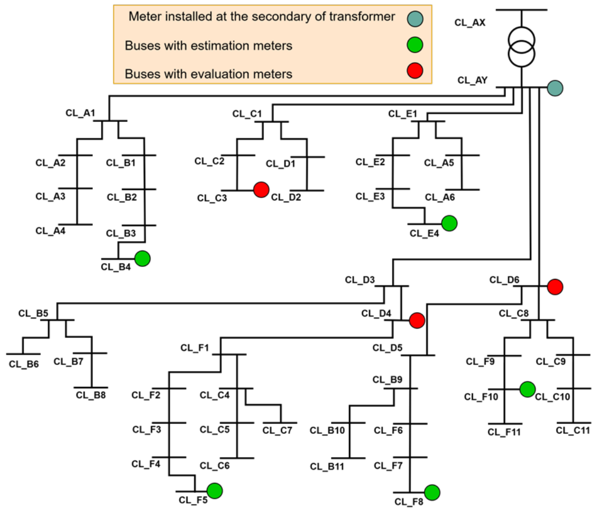


Figure 1: Location of metering devices used for digital twin development and evaluation

estimation meters, whose three-phase voltage measurements are directly fed into the DSSE algorithm, are indicated by green markers. In total, 5 nodes are instrumented with estimation meters. Additionally, 3 nodes are equipped with evaluation meters, shown by red markers. Measurements from these nodes are excluded from the estimation process and are solely used for post-estimation performance evaluation. By intentionally withholding measurements from a subset of nodes, the proposed setup enables a fair and unbiased assessment of the estimation accuracy. This measurement configuration closely reflects practical distribution grid scenarios, where monitoring infrastructure is limited and full observability is rarely available. Consequently, only a subset of nodes is equipped with meters, making the DSSE problem both challenging and representative of real-world three-phase unbalanced distribution systems.

2.2 Problem Definition and Solution Method

The problem addressed in this study is the estimation of nodal voltage states in a low-voltage distribution grid operating under limited observability. Accurate voltage estimation requires reliable knowledge of nodal power injections; however, such quantities are not directly measurable at all nodes due to sparse metering infrastructure. To overcome this limitation, nodal active and reactive power injections are inferred through short-term forecasting based on historical measurements. Specifically, the problem is formulated as a 24-hour-ahead prediction task in which the net active and reactive power at each node are estimated as $\hat{P}_{t+24}, \hat{Q}_{t+24}$.

Although the forecasting framework produces predictions over the full 24-hour horizon, only the terminal forecasted time step is used in the subsequent phase allocation and voltage state estimation stages. This choice reflects the focus on the most imminent operating condition, which is critical for real-time digital twin operation and network monitoring.

The dataset used in this study comprises historical power measurements at each node of the low-voltage distribution grid. Unlike standard load datasets that provide net power directly, each node contains four separate time series:

- Active power imported, P_t^{imp},
- Active power exported, P_t^{exp},
- Reactive power imported, Q_t^{imp},
- Reactive power exported, Q_t^{exp}.

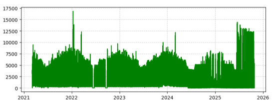
The net active and reactive power at each time step is computed as:

$$P_t = P_t^{\text{imp}} - P_t^{\text{exp}}, \tag{1}$$

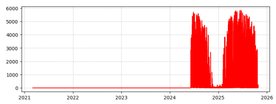
$$Q_t = Q_t^{\text{imp}} - Q_t^{\text{exp}}. \tag{2}$$

Figure 2 illustrates the disaggregated imported and exported power for node CL-B3.

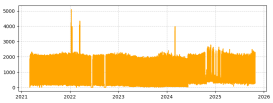
The solution methodology will be detailed in the following section, which is organized as follows: first, the aggregated nodal power data are forecasted using the load forecasting method; next, the predicted values are disaggregated into individual phases using a phase-level power disaggregation procedure; and finally, the hybrid optimization algorithm is applied for voltage state estimation across the network, as described in Section 3.



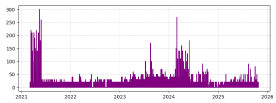
(a) Import Active Power



(b) Export Active Power



(a) Import Reactive Power



(b) Export Reactive Power

Figure 2: Import and Export Active/Reactive Power for CL-B3

3 Proposed Methodology

In this section proposed framework including forecasting, disaggregation, and hybrid meta-heuristic optimization are discussed.

3.1 Load Forecasting Method

Recently, we proposed two forecasting methods for energy applications and grid optimization, which have demonstrated enhanced prediction accuracy and robustness [15, 16]. In this work, we employ a modified version of these methods. Prior to forecasting, the data undergoes the following preprocessing steps:

(1) Conversion of the time index into a continuous hourly sequence, inserting any missing timestamps.
(2) Aggregation of multiple measurements at the same timestamp by computing their mean.
(3) Handling potential missing measurements using time-based interpolation with forward- and backward-filling to ensure continuity and numerical stability.

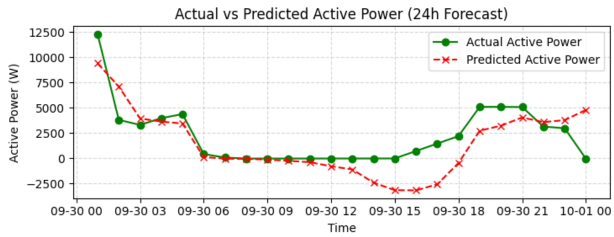
To improve stability and accuracy beyond individual models, a multi-level ensemble approach [16]was employed using the Voting Regressor framework. The final predictor combines models with complementary strengths:

- Decision Trees,
- Random Forests,
- XGBoost regressors,
- Auxiliary internal ensembles for deeper averaging.

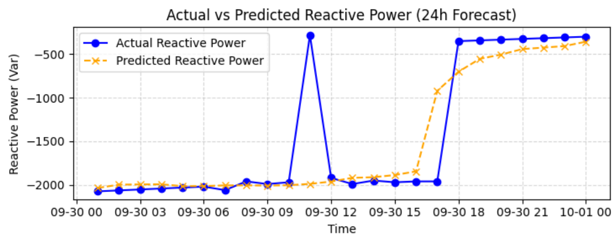
The ensemble aggregates the outputs of all base models via weighted averaging:

$$\hat{y}_t = \sum_{i=1}^{M} w_i \hat{y}_t^{(i)}, \tag{3}$$

where $\hat{y}_t^{(i)}$ is the prediction of model i and w_i its corresponding weight. This multi-level ensemble produces reliable 24-hour-ahead forecasts for both active and reactive power. Each node's time series is processed independently, preserving node-specific accuracy and capturing localized consumption patterns. Figure 3 shows the predictions for active and reactive power by the proposed forecast Method for the next 24 hours.



(a)



(b)

Figure 3: Predicted Active and Reactive Power by the proposed forecast method

3.2 Phase-Level Power Disaggregation

Total forecasted active and reactive power at each bus is disaggregated into the three phases $\{A, B, C\}$ using a phase-level power disaggregation procedure. Phase ratios, representing the proportion of power allocated to each phase, are first computed from the meter measurements. These ratios are subsequently assigned to unmetered nodes through impedance-based clustering, ensuring that electrically similar nodes exhibit comparable phase distributions. Let $\hat{P}_{\text{tot}}$ and $\hat{Q}_{\text{tot}}$ denote the predicted total active and reactive powers at a given node. The phase-level active and reactive power estimates at each cluster, denoted by $\left(\hat{P}_A, \hat{P}_B, \hat{P}_C\right)$ and $\left(\hat{Q}_A, \hat{Q}_B, \hat{Q}_C\right)$, respectively, are obtained by scaling the total power with the corresponding phase ratios. The resulting phase-level powers satisfy the power balance constraints

$$\hat{P}_A + \hat{P}_B + \hat{P}_C = \hat{P}_{\text{tot}}, \tag{4}$$

$$\hat{Q}_A + \hat{Q}_B + \hat{Q}_C = \hat{Q}_{\text{tot}}. \tag{5}$$

The resulting phase-level power injections are then used as inputs to the voltage state estimation process.

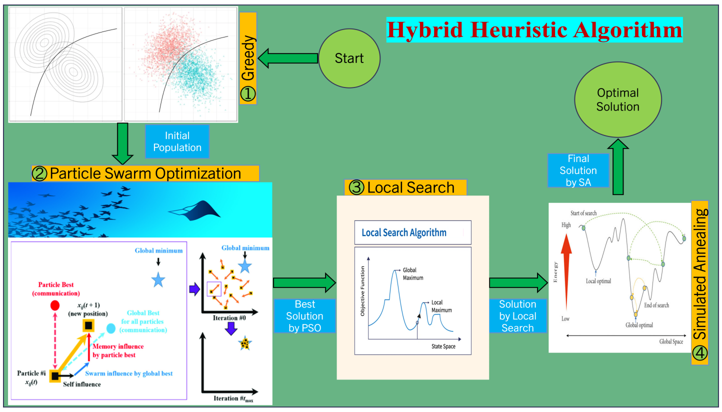


Figure 4: Workflow of the Hybrid Optimization Algorithm for Voltage State Estimation

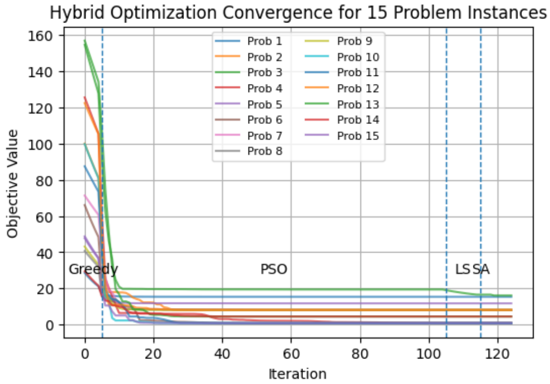


Figure 5: Convergence behavior of the proposed Greedy–PSO–LS–SA framework over 15 similar problem instances.

3.3 Hybrid Optimization-Based Voltage State Estimation

After obtaining the total active and reactive power at each node, the next step is to estimate the nodal voltages across the distribution grid. To achieve this, a hybrid optimization algorithm is developed, sequentially integrating greedy heuristics, PSO, local search, and simulated annealing (SA).

3.3.1 Objective Function. The problem is formulated as an optimization task that minimizes the deviation between the estimated nodal voltages, obtained after load adjustments through OpenDSS, and the reference voltages measured at selected nodes. Let V_i^{est} denote the estimated voltage magnitude at node i and V_i^{ref} the corresponding reference measurement. The objective function is defined as

$$\min_{\Delta P, \Delta Q} \sum_{i \in \mathcal{N}} \left(V_i^{\text{est}}(\Delta P, \Delta Q) - V_i^{\text{ref}} \right)^2, \tag{6}$$

where $\mathcal{N}$ denotes the set of monitored nodes, and ΔP and ΔQ represent the active and reactive power adjustments applied to the loads.

3.3.2 Hybrid Optimization Procedure. The algorithm proceeds through the following stages:

(1) **Greedy Heuristic:** Iteratively adjusts loads in small increments to locally minimize the objective function, providing a feasible starting point for global optimization.
(2) **Particle Swarm Optimization (PSO):** Uses the greedy solution to initialize particle positions. PSO explores the solution space to identify improved load adjustments, updating each particle based on personal and global best scores.
(3) **Local Search (LS):** Refines the PSO solution by exploring load adjustments along directions indicated by influence factors, fine-tuning the solution around the optimum.
(4) **Simulated Annealing (SA):** Introduces stochastic exploration to escape potential local minima, gradually reducing the acceptance probability of worse solutions until convergence.

Figure 4 illustrates the workflow and key steps of the hybrid optimization algorithm. Figure 5 illustrates the convergence behavior of the proposed hybrid optimization framework over fifteen independent problem instances. Despite different initial conditions, the algorithm exhibits consistent behavior, with the greedy heuristic providing rapid objective reduction followed by stable refinement through PSO, local search, and simulated annealing. By combining these techniques, the hybrid approach leverages their complementary strengths: the greedy heuristic identifies promising regions, PSO explores global optima, local search fine-tunes the solution, and SA ensures robustness against local minima. The resulting algorithm produces high-accuracy voltage estimates, which are critical for reliable grid monitoring and control.

The output consists of optimal adjustments for active and reactive power at each load. Applying these adjustments generates voltage profiles that closely match the reference measurements, effectively balancing global exploration and local exploitation for accurate state estimation in large-scale low-voltage networks.

4 Results and Discussion

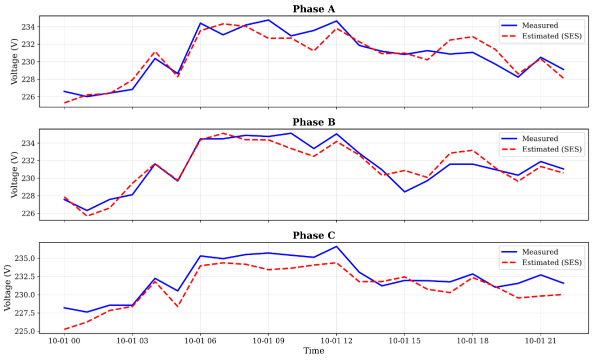
In this section results and discussion including state estimation results, evaluation of the proposed framework, and the effect of changing in the meta-heuristic initialization are proposed.

4.1 State Estimation Results

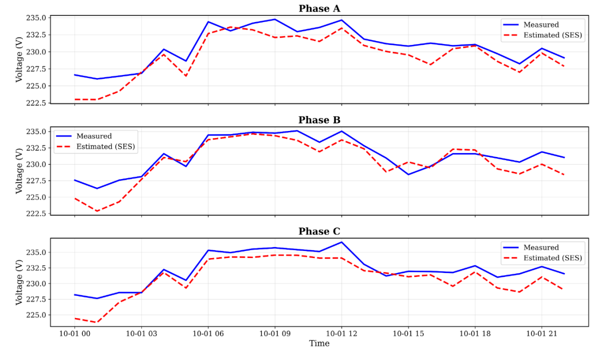
Figure 6 presents a comparison between measured and estimated voltage profiles for the three phases (A, B, C) at the selected evaluation nodes over the study period. Each sub-figure -(a), (b), (c)-corresponds to a specific estimation node, with the three subplots illustrating the voltage magnitudes of phases A, B, and C, respectively. The solid blue curves represent the measured voltages acquired from metering devices, while the red dashed curves denote the voltages estimated using the proposed framework. The results demonstrate that the proposed framework effectively captures the temporal variations of voltage magnitudes across all three phases. The strong agreement between measured and estimated voltage profiles confirms the capability of the optimization-based state estimation approach to accurately reconstruct system states under limited observability conditions. The small discrepancies observed between measured and estimated voltages can be attributed to inherent uncertainties in load forecasting, phase disaggregation, and approximation errors introduced during the optimization process.

4.2 Evaluating of the Proposed Method

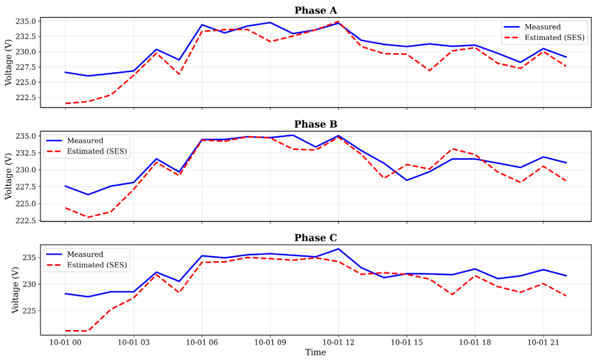
In this subsection, the performance of the proposed hybrid optimization pipeline is evaluated. The greedy heuristic is first employed to rapidly generate an initial feasible solution. Its primary role is to reduce the search space and provide a computationally



(a) Bus CL-C3 for three phases (A, B, C)



(b) Bus CL-D6 for three phases (A, B, C)



(c) Bus CL-D4 for three phases (A, B, C)

Figure 6: Comparison of measured and state-estimated voltages

efficient starting point for the subsequent optimization stages. Table 2 reports the performance metrics obtained using the proposed framework with greedy initialization.

Table 2: Performance Metrics with Greedy Initialization

Metric	Greedy Initialization
Avg. Runtime per Step (s)	290.91
Total Runtime (s)	6690.85
RMSE (V)	1.1558
MAE (V)	0.9084
Std. Deviation (V)	1.0598
Max Error (V)	4.1240

As shown in Table 2, the greedy initialization achieves relatively low estimation errors, demonstrating its effectiveness in quickly identifying a reasonable solution. However, this comes at the cost of a higher computational burden, as reflected in the average runtime per step and the total runtime. While the greedy heuristic initialization provides acceptable accuracy, it lacks the low computational complexity. Figure 7 further illustrates the distribution of voltage estimation errors.

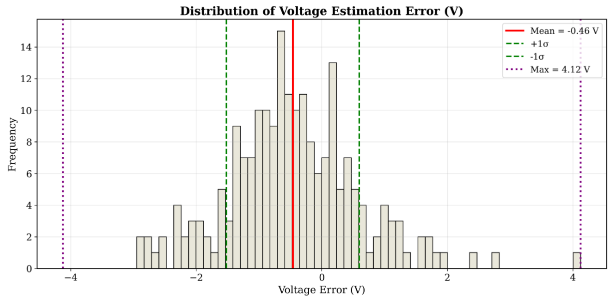


Figure 7: Histogram of voltage estimation error with mean, standard deviation, and maximum error

4.3 Impact of Changing SE Initialization

As shown in Table 3, employing a warm-start strategy, where the solution from the previous time interval is used to initialize of the optimization method instead of a random initialization by greedy, the average runtime per step and the total runtime are both reduced by approximately 54.9% compared to the baseline. This significant improvement highlights the effectiveness of the warm-start initialization in accelerating convergence. However, this computational gain comes at the expense of estimation accuracy. The RMSE and MAE increase by 35.7% (0.41 V) and 36.3% (0.32 V), respectively, while the standard deviation and maximum error rise by 29.0% (0.3 V) and 26.3% (1.09 V). These results indicate that the observed performance trade-off is primarily attributable to the efficiency-oriented nature of the warm-start method, rather than the greedy mechanism itself. In other words, the warm-start initialization prioritizes faster convergence over solution refinement, which explains the reduced accuracy despite the substantial runtime improvement.

Table 3: Performance Metrics Using Warm Start

Metric	Warm Start
Avg. Runtime per Step (s)	131.13
Total Runtime (s)	3015.98
RMSE (V)	1.5689
MAE (V)	1.2382
Std. Deviation (V)	1.3673
Max Error (V)	5.2109

5 Conclusion

This paper presented a three-stage optimization-based framework by integrating a short-term nodal load forecasting, phase-level power disaggregation, and a hybrid optimization algorithm for voltage estimation in low-voltage distribution grids. Voltage estimation is formulated as a network optimization problem and solved using a hybrid metaheuristic combining greedy initialization, particle swarm optimization, local search, and simulated annealing. The proposed approach achieves accurate voltage estimates with manageable computational complexity. The results demonstrate that coupling intelligent forecasting with hybrid optimization enhances grid observability and estimation accuracy, while the modular framework enables seamless integration into digital twin platforms for near–real-time monitoring and decision-making. Future work will focus on adaptive tuning of the hybrid optimization parameters, allowing the algorithm to automatically adjust exploration and exploitation strategies in response to network size, operating conditions, and convergence behavior. In addition, uncertainty-aware formulations will be incorporated to explicitly account for forecasting errors and measurement noise, enabling more robust voltage estimation through stochastic or robust optimization techniques.

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