Short Paper

An Extended Formulation for the Constrained Routing and Spectrum Assignment Problem in Elastic Optical Networks

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ABSTRACT

The Routing and Spectrum Assignment problem consists of routing a given set of origin-destination traffic demands and assigning them to contiguous spectrum frequencies such that no frequency slot is assigned to more than one demand within a network link. This work deals with the variant where each demand route must additionally satisfy a maximal-length constraint. In this paper we propose a compact extended formulation for the Constrained Routing and Spectrum Assignment Problem. We show that our extended formulation is stronger than formulations known in the literature. Experimental results demonstrate the efficiency of our approach.

KEYWORDS

optical networks, routing and spectrum assignment problem, extended formulations, combinatorial optimization

1 INTRODUCTION

Modern optical networks represent a crucial infrastructure for our information society. To transmit signals, light is used as a communication medium between sending and receiving nodes. For over two decades, *Wavelength-Division Multiplexing (WDM)* has been the most popular technology used in optical networks, where different wavelengths are used to simultaneously transmit signals over a single optical fiber. Hereby, the wavelengths have to be selected from a rather coarse fixed grid of frequencies specified by the United Nations agency ITU (International Telecommunication Union), which leads to an inefficient use of spectral resources.

In response to the continuous growth of data traffic volumes in communication networks, a new generation of optical networks, called Elastic Optical Networks (EONs), has been introduced to enhance the spectrum efficiency and to enlarge the network capacity [10]. In EONs, the frequency spectrum of an optical fiber is divided into many narrow frequency slots, and any sequence of contiguous slots can form a channel to create an optical connection between sending and receiving nodes, called lightpath. That way, EONs enable capacity gain by allocating minimum required bandwidth to every traffic demand thanks to a finer spectrum granularity than in the traditional WDM networks.

To operate EONs, the so-called *Routing and Spectrum Assignment (RSA) problem* has to be solved which consists of establishing the lightpaths (represented by a route and a channel) for a set of traffic demands (given as sending and receiving nodes and required slot numbers), thereby optimizing some objective function. To comply with ITU regulations, the following constraints need to be respected when dealing with the RSA problem:

(1) *spectrum continuity*: the frequency slots allocated to a demand remain the same on all the links of a route;
(2) *spectrum contiguity*: the frequency slots allocated to a demand must be contiguous;
(3) *non-overlapping spectrum*: on each link of the network, a frequency slot can be allocated to at most one demand.

In addition, technical properties further force that the length of a route must not exceed the transmission reach of the optical signal which leads to the *Constraint Routing and Spectrum Assignment (CRSA) problem*, see Section 2 for details.

The RSA problem has started to receive a lot of attention over the last few years. It has been shown to be NP-hard [4, 19]. In fact, if for each demand the route is already known or uniquely determined (e.g. if the optical network is a tree), then the RSA problem reduces to the spectrum assignment (SA) problem and only consists of determining the demand's channels. It is NP-complete to decide whether there is a feasible spectrum assignment within a given optical spectrum, even if the optical network is a path, see e.g. [17]. This makes the RSA problem much harder than the WDM problem which is polynomially solvable on paths, see e.g. [7].

To solve the RSA problem, various approaches have been studied in the literature, based on different Integer Linear Programming (ILP) models. Hereby, detailed models aiming at precisely describing all technological aspects of EONs and being able to handle various criteria for optimization typically suffer from tractability issues resulting from their greater complexity such that the tendency is to use simplified or restricted models.

The majority of the existing models uses an *edge-path formulation* where for each demand, variables are associated either with all possible routing paths or with all possible lightpaths for this demand. One characteristic of this formulation is, therefore, an exponential number of variables issued from the number of feasible paths between origin-destination pairs in the network, which grows exponentially with the size of the network and the number of demands.

To bypass the exponential number of variables, edge-path formulations with a precomputed subset of all possible paths per

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demand have been studied e.g. in [11, 13, 18], see [20] for an overview. However, such formulations cannot guarantee optimality of the solutions in general (as only a precomputed subset of paths is considered and, thus, a restricted problem solved). In order to be able to find optimal solutions of the RSA problem w.r.t. any objective function with the help of an edge-path formulation, all possible paths have to be taken into account. As the explicit models are far too big for computation, it is in order to apply column-generation methods. However, computational results from e.g. [12, 14, 16] show that the size of the instances that can be solved that way is rather limited [1] .

An alternative to edge-path formulations is to use *edge-node formulations* that have the advantage to be compact in terms of the number of variables and constraints, but have the disadvantage that the routing is rather involved and less intuitive. Only few authors made use of this type of model, as Cai et al. [3], Velasco et al. [18], Zotkiewiez et al. [20], and Jia et al. who used in [9] an edge-node formulation to treat a more general problem. As noticed in [8], the models from [3, 18, 20] are incomplete as their feasible region is a superset of all feasible solutions of the RSA problem and can, thus, handle only some objective functions. The first complete edge-node formulation presented in [8] exactly encodes the feasible solutions, but requires an exponential number of constraints to ensure proper routings. Moreover, in [8] a procedure is given to separate the exponentially-sized families of constraints in polynomial time, which makes the model computationally competitive with the compact but incomplete models from [3, 18, 20].

The computational tests in [8] were performed on a set of instances that resembles those used in [3, 18, 20]. However, the used instances are rather small and it is expected that the running time to optimally solve real-sized instances of the RSA problem will be drastically increased with all existing edge-node formulations.

Our contribution is to develop the edge-node formulations from [8] further to a compact model that enables solving real-sized instances of the CRSA problem in reasonable time. For that, we propose an extended formulation based on the works from [1]. Our model is applied over an auxiliary network constructed from the original optical network and requires an appropriate modification of the involved variables to simplify the way to encode routings, see Section 3. We provide the relations binding the variables of the original formulation to the ones in our model. In Section 4, we show that from a theoretical point of view, our extended formulation is stronger than the formulation from [8]. In Section 5, experimental results demonstrate the efficiency of our approach. We close with some concluding remarks and lines of future research.

2 PROBLEM DEFINITION

In this section, we formally define the CRSA problem by describing in detail the input and the desired output of the CRSA problem together with the studied objective functions.

As input of the CRSA problem, we are given

- an optical spectrum $S = \{1, \dots, \bar{s}\}$ of available frequency slots;
- an optical network, represented as an undirected, loopless, connected graph $G = (V, E)$ that may have parallel edges (if parallel optical fibers are installed between two nodes), and for each edge $e \in E$ its length $\ell_e \in \mathbb{R}_+$,
- a multiset K of demands where each demand $k \in K$ is specified by
  - an origin node $o_k \in V$ and a destination node $d_k \in V \setminus o_k$,
  - a requested number $w_k \in \mathbb{N}_+$ of slots, and
  - a transmission reach $\bar{\ell}_k \in \mathbb{R}_+$.

The task is to determine for each demand $k \in K$ a lightpath composed of an (o_k, d_k)-path P_k in G respecting the transmission reach $\bar{\ell}_k$ and a channel $S_k \subset S$ of w_k consecutive frequency slots (*spectrum contiguity*) that is available on all edges of P_k (*spectrum continuity*) and disjoint from the channels $S_{k'}$ of all other demands $k' \in K$ routed along an edge of P_k (*non-overlapping spectrum*), thereby minimizing some objective function. Note that we focus on the case where all demands must be satisfied.

Hence, the desired output of the CRSA problem is, for each demand $k \in K$, a lightpath composed of

- an (o_k,d_k)-path P_k in G with $\sum_{e \in E(P_k)} l_e \leq \bar{\ell}_k$,
- a subset $S_k \subset \{1, \dots, \bar{s}\}$ of w_k consecutive slots with $S_k \cap S_{k'} = \emptyset$ for each demand $k' \in K$ routed along an edge $e \in E(P_k)$.

In addition, the selected set of lightpaths is supposed to minimize a chosen objective function, for instance:

O_1: minimize the sum of hops in paths (where the term hops refers to the number of edges in a path P_k),
O_2: minimize the sum of the total length of paths (taking the edge weights ℓ_e into account),
O_3: minimize the maximal used slot position (and, thus, the width of the subspectrum of S used for the spectrum assignment),
O_4: minimize the sum of the maximal used slot positions over all demands.

Note that the two objective functions O_1 and O_2 are only related to the routing (provided that a feasible spectrum assignment within S exists for this routing), whereas the other two objective functions O_3 and O_4 seek for the most efficient spectrum assignments over all possible routings.

3 EXTENDED FORMULATION

Our extended formulation is based on the ILP formulation proposed by [8], hereafter denoted by RSA-BASE. This formulation mainly uses the following three sets of binary variables (other variables may be added according to the choice of the objective function) in order to model the RSA problem. For each demand $k \in K$ and each edge $e \in E$, variables $x^k_e \in \{0, 1\}$ are used to indicate whether or not demand k is routed through edge e. For each demand $k \in K$ and each slot $s \in S$, variables $z^k_s \in \{0, 1\}$ express the fact of whether or not the slot s is the last slot of the channel assigned to demand k. Finally, for each demand $k \in K$, each slot $s \in S$ and each edge $e \in E$, variables $t^{sk}_e \in \{0, 1\}$ indicate whether or not demand k uses the slot s on edge E. In the case where all demands must be satisfied, the formulation RSA-BASE

[1] An exception are edge-path formulations from [5, 6] that seem to be scalable to real-size instances by using column-generation methods. However, the authors of [5, 6] consider an asymmetric version of the RSA problem where each link of the optical network is composed by two optical fibers to be used to transmit signals in one direction only. This makes the spectrum assignment easier (as less restrictions have to be taken into account), but is not used very often in practice by network operators as that way it is not possible to use the full spectral resources of the optical links.

from [8] reads as follows:

$$\sum_{e\in\delta(v)} x_e^k = 1, \qquad \forall k \in K, v \in \{o_k, d_k\}, \tag{1}$$

$$\sum_{e\in\delta(X)} x_e^k \geq 1, \qquad \forall k, X \subseteq V \setminus d_k, o_k \in X, \tag{2}$$

$$\sum_{e\in\delta(v)} x_e^k \leq 2, \qquad \forall k \in K, v \in V \setminus \{o_k, d_k\}, \tag{3}$$

$$\sum_{e\in\delta(X)} x_e^k \geq 2x_{uv}^k \qquad \forall k, X \subset V, u, v \in X, o_k, d_k \notin X \tag{4}$$

$$\sum_{e\in\delta(X)} x_e^k \geq x_{uv}^k \qquad \forall k, X \subset V, u, v \in X, o_k \text{ or } d_k \in X \tag{5}$$

$$\sum_{e\in E} l_e x_e^k \leq \bar{\ell}_k \qquad \forall k \in K, \tag{6}$$

$$\sum_{w_k \leq s \leq \bar{s}} z_s^k = 1, \qquad \forall k \in K, \tag{7}$$

$$\sum_{1\leq s< w_k} z_s^k = 0, \qquad \forall k \in K, \tag{8}$$

$$\sum_{s\in S} t_e^{s,k} = w_k x_e^k, \qquad \forall k \in K, e \in E, \tag{9}$$

$$x_e^k + \sum_{s'=s}^{\min(s+w_k-1,\bar{s})} z_{s'}^k \leq 1 + t_e^{s,k} \qquad \forall k \in K, e \in E, s \in S, \tag{10}$$

$$\sum_{k\in K} t_e^{s,k} \leq 1, \qquad \forall e \in E, s \in S, \tag{11}$$

$$x_e^k, z_s^k, t_e^{s,k} \in \{0,1\} \qquad \forall k \in K, e \in E, s \in S. \tag{12}$$

To select a route for each demand $k \in K$, the origin/destination constraint (1) ensures that one path leaves o_k and enters d_k, the path-continuity constraints (2) guarantee that an edge crosses the cut $(X, V \setminus X)$ for each X with $o_k \in X$ and $d_k \in V \setminus X$, the degree constraints (3) prevent cycles attached to a route whereas the cycle-elimination constraints (4) and (5) exclude cycles isolated from the route, and the constraints (6) ensure that the transmission reach of the route is respected.

To select for each demand $k \in K$ a channel $S_k \subset S$ of w_k consecutive frequency slots, the channel selection constraints (7) force to select one slot as last slot of S_k whereas constraints (8) exclude the slots $s < w_k$ from being the last slot of S_k, the edge-slot constraints (9) allocate w_k slots to demand k on edge e whenever k is routed through e, constraints (10) ensure spectrum contiguity and continuity, and the non-overlapping constraints (11) guarantee that a slot s on edge e can be allocated to at most one demand.

The extended formulation proposed in this paper consists of constructing a directed graph G' from the network graph G as well as combining these three sets of variables into a single set of flow variables. The directed graph G' is constructed by simply creating an in-going arc (u, v) and an out-going arc (v, u) for each edge uv in $E(G)$. Then for each demand $k \in K$, each slot $s \in S$ and each arc $a \in A(G')$, we define the binary variable f_a^{sk} as follows.

$$f_a^{sk} = \begin{cases} 1, & \text{if } k \text{ is routed through arc } a \text{ and } s \text{ is its last channel slot;} \\ 0, & \text{otherwise.} \end{cases}$$

It follows that every variable x, z and t from the formulation RSA-BASE can be described as a linear function of the above defined variables f. Indeed, we have that

$$x_{uv}^k = \sum_{s=1}^{\bar{s}} \left(f_{(u,v)}^{sk} + f_{(v,u)}^{sk} \right) \qquad \forall uv \in E, k \in K, \tag{13}$$

$$z_s^k = \sum_{a\in\delta^+(o_k)} f_a^{sk} \qquad \forall k \in K, s \in S, \tag{14}$$

$$t_{uv}^{sk} = \sum_{i=s}^{\min\{\bar{s},s+w_k-1\}} \left(f_{(u,v)}^{ik} + f_{(v,u)}^{ik} \right) \quad \forall uv \in E, k \in K, s \in S. \tag{15}$$

By applying this variable transformation we obtain the extended formulation hereafter denoted RSA-EXT. That way, we can now perceive the RSA as a multi-commodity flow problem where each commodity is associated with a demand.

Notice that while formulation RSA-BASE requires $\bar{s}|K||E| + \bar{s}|K| + |E||K|$ variables, formulation RSA-EXT requires $2(\bar{s}|K||E|)$ variables. Therefore RSA-EXT uses less than twice the number of variables employed in RSA-BASE. Moreover, RSA-BASE and RSA-EXT use the same number of constraints. Finally, RSA-BASE and RSA-EXT are equally strong in the sense that the optimal solutions of their linear relaxations have, by definition, the same value.

4 FORMULATION IMPROVEMENT

In this section we describe how formulation RSA-EXT can be reinforced and hence stronger than RSA-BASE.

4.1 Reinforcement of length constraints

In RSA-BASE, the maximum reach of a demand routed path is ensured through the following length constraints

$$\sum_{e\in E(G)} l_e x_e^k \leq \bar{\ell}_k \qquad \forall k \in K,$$

which become

$$\sum_{a\in A(G')} \sum_{s=1}^{\bar{s}} l_a f_a^{sk} \leq \bar{\ell}_k \qquad \forall k \in K, \tag{16}$$

with the variable transformation described in Section 3. Consider now the following *disaggregated length inequalities.*

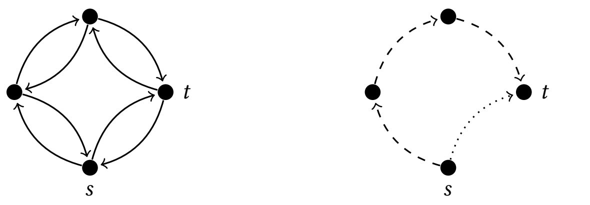
$$\sum_{a\in A(G')} l_a f_a^{sk} \leq \bar{\ell}_k \sum_{a\in\delta^+(o_k)} f_a^{sk} \qquad \forall k \in K, s \in S. \tag{17}$$

Proposition 4.1. *The disaggregated length inequalities* (17) *are valid inequalities.*

Proof. For a given demand k and slot s, $0 \leq \sum_{a\in\delta^+(o_k)} f_a^{sk} \leq 1$ since only one path is to be found for routing k. If $\sum_{a\in\delta^+(o_k)} f_a^{sk} = 0$, then demand k cannot be routed through channel $\{s - w_k + 1, \ldots, s\}$ and hence variable f_a^{sk} must equal 0 for any $a \in A(G')$. In return, if $\sum_{a\in\delta^+(o_k)} f_a^{sk} = 1$, then the path used for routing demand k respects its maximum reach $\bar{\ell}_k$. □

Notice that by summing up inequalities (17) for each $s \in S$ one obtains the original length constraints (16). Moreover, it is easy to see that there exists some fractional solutions that satisfy the original inequalities (16) but violate the proposed inequalities (17). The following example illustrates such a situation.

Consider the following RSA instance where the network G' is depicted in Figure 1a and all arcs have length 1. Let $\bar{s} = 2$, and let K be composed of a single demand k going from node s to node t, $w_k = 1$ and $\bar{\ell}_k = 2$. Clearly the proposed fractional solution verifies the original length constraints (16) but is cutoff by inequalities (17). For this reason, we replace the original length constraints (16) by the proposed disaggregated length constraints (17) in the RSA-EXT formulation.



(a) Network G'. **(b) Solution illustration.**

Figure 1: Illustration of a fractional solution satisfying constraints (16) and violating constraints (17). In (b), all represented arcs are associated with 0.5 valued variables, the dashed path uses channel $\{1\}$, the dotted path channel $\{2\}$.

4.2 Making the extended formulation compact

In order to ensure path-continuity (*i.e.*, the fact that the path leaving o_k reaches d_k for each demand $k \in K$) and to eliminate any cycle in the demand route, formulation RSA-BASE uses an exponential number of constraints. Next, we show how in RSA-EXT these constraints can either be dropped or replaced with a polynomial number of inequalities in order to obtain a compact extended formulation.

4.2.1 Ensuring path continuity. The demand's path continuity is ensured in RSA-BASE through the constraints (2)

$$\sum_{e \in \delta(X)} x_e^k \geq 1, \qquad \forall k \in K, X \subseteq V \setminus d_k, o_k \in X,$$

which become

$$\sum_{a \in \delta^+(X)} \sum_{s=1}^{\bar{s}} f_a^{sk} + \sum_{a \in \delta^-(X)} \sum_{s=1}^{\bar{s}} f_a^{sk} \geq 1, \qquad \forall k \in K, X \subseteq V, \quad o_k \in X, d_k \notin X, \tag{18}$$

with the variable transformation proposed in Section 3. Consider now the following *classic flow conservation constraints.*

$$\sum_{a \in \delta^+(v)} \sum_{s=1}^{\bar{s}} f_a^{sk} - \sum_{a \in \delta^-(v)} \sum_{s=1}^{\bar{s}} f_a^{sk} = \begin{cases} 1, & \text{if } v = o_k \\ 0, & \text{if } v \in V \setminus \{o_k, d_k\} \\ -1, & \text{if } v = d_k \end{cases} \quad \forall k \in K, v \in V. \tag{19}$$

Proposition 4.2. *The classic flow conservation constraints* (19) *are valid.*

Lemma 4.3. *The path continuity constraints* (18) *are dominated by the classic flow conservation constraints* (19).

Proof. For a given subset $X \subseteq V \setminus \{d_k\}, o_k \in X$, and a demand $k \in K$, summing up equations (19) for each $v \in X$ yields the equation

$$\sum_{a \in \delta^+(X)} \sum_{s=1}^{\bar{s}} f_a^{sk} - \sum_{a \in \delta^-(X)} \sum_{s=1}^{\bar{s}} f_a^{sk} = 1.$$

Since every variable is required to be nonnegative, we have

$$\sum_{a \in \delta^+(X)} \sum_{s=1}^{\bar{s}} f_a^{sk} \geq 1.$$

It follows directly that for any $k \in K, X \subseteq V$ such that $o_k \in X$ and $d_k \notin X$, the following holds

$$\sum_{a \in \delta^+(X)} \sum_{s=1}^{\bar{s}} f_a^{sk} + \sum_{a \in \delta^-(X)} \sum_{s=1}^{\bar{s}} f_a^{sk} \geq \sum_{a \in \delta^+(X)} \sum_{s=1}^{\bar{s}} f_a^{sk} \geq 1,$$

which concludes the proof. □

Consider now the following *disaggregated flow conservation constraints* (20).

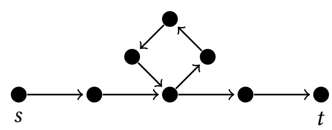
$$\sum_{a \in \delta^+(v)} f_a^{sk} - \sum_{a \in \delta^-(v)} f_a^{sk} = 0 \;\; \forall k \in K, v \in V \setminus \{o_k, d_k\}, s \in S. \tag{20}$$

Proposition 4.4. *The disaggregated flow conservation constraints* (20) *are valid.*

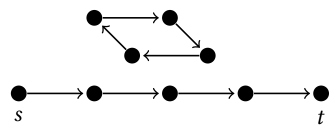
Notice that the disaggregated flow constraints (20) are stronger than the initially proposed flow constraints (19). Indeed, summing up constraints (20) for $s \in S$ yields the former constraints (19). For this reason, we replace the path continuity constraints (18) by the disaggregated flow constraints (20) in RSA-EXT formulation.

4.2.2 Dropping cycle-elimination constraints. In order to prevent cycles in the route of the demands, RSA-BASE includes an exponential number of cycle-elimination constraints. Next we show that for the objective functions studied in this paper, such constraints can be dropped, and whenever a solution including cycles is found, a simple post-processing is capable of finding a solution of at most the same cost without any cycles in polynomial time.

Cycles may appear in a demand route either attached to it or detached from it (see Figure 2). For cycles attached to the route (*e.g.*, Figure 2a), including a polynomial sized family of *degree constraints* suffices for excluding such solutions.



(a) The flow path of an (s, t)-demand with a cycle attached to it.



(b) The flow path of an (s, t)-demand with a detached cycle.

Figure 2: Possible flow structures that can be found by dropping cycle-elimination constraints.

Proposition 4.5. *The following degree constraints are valid:*

$$\sum_{a \in \delta^+(v)} \sum_{s=1}^{\bar{s}} f_a^{sk} \leq 1 \qquad \forall k \in K, v \in V. \tag{21}$$

For excluding cycles that are detached from the demand's route (*e.g.*, Figure 2b), an exponential number of inequalities is used in formulation RSA-BASE. For the objective functions studied here however such inequalities can be dropped, making formulation RSA-EXT compact. The next proposition describes how to obtain a feasible solution from a solution with cycles. The obtained solution has at most the same cost as the cyclic one.

Proposition 4.6. *Given a linear objective function where every variable f_a^{sk} implies a cost $c_a^{sk} \geq 0$, any solution to the RSA problem containing cycles can be transformed into an acyclic solution of at most the same cost as the former solution.*

Proof. Let $\bar{f}$ be a solution of the RSA-EXT formulation where the route of a given demand $k' \in K$ includes a cycle detached from its main path (*e.g.*, Figure 2b). Let C denote the set of arcs composing a cycle in the route of demand k'. Then, since every variable cost c_a^{sk} and every arc length is nonnegative, the solution $\hat{f}$ defined as

$$\hat{f}_a^{sk} = \bar{f}_a^{sk} \qquad \forall k \in K \setminus k', a \in A(G'), s \in S,$$

$$\hat{f}_a^{sk'} = \bar{f}_a^{sk'} \qquad \forall a \in A(G') \setminus C, s \in S,$$

$$\hat{f}_a^{sk'} = 0 \qquad \forall a \in C, s \in S,$$

is a feasible solution without cycle C that costs at most the cost of solution $\bar{f}$. □

It follows from Proposition 4.6 that whenever RSA-EXT yields a solution containing cycles, a postprocessing method is capable of providing an acyclic solution of at most the same cost in polynomial time. Such postprocessing involves a simple labeling algorithm. Our formulation is therefore compact and solves the RSA problem for all the objective functions studied.

4.3 Variable elimination

In order to boost the performances of the proposed formulation, a preprocessing method for eliminating variables may be applied.

Proposition 4.7. *If, for a given demand $k \in K$, the shortest path from o_k to d_k passing through arc $a \in A(G')$ has length strictly greater than $\bar{\ell}_k$, then*

$$f_a^{sk} = 0 \qquad \forall s \in S. \tag{22}$$

Computing the shortest path between two nodes s and t passing through an arc (u, v) can be done in polynomial time by computing the shortest path from s to u and from v to t. In order to identify the variables in Proposition 4.7 that can be eliminated, we apply this preprocessing for every demand $k \in K$ and every arc $(u, v) \in A(G')$. Such preprocessing is hereafter denoted as *length preprocessing*.

Since G is supposed to be connected, there always exists a path between any two given nodes in G'. However, when applying the variable elimination described in Proposition 4.7, one might trouble this property. Let $D_k \subseteq A(G')$ denote the set of arcs that are unable to route demand k due to length restrictions (*i.e.*, the arcs identified through length preprocessing). Now, consider the practical graph for demand $k \in K$ defined as $G'_k = (V(G'), A(G') \setminus D_k)$. Graph G'_k is not necessarily a connected graph and hence the following proposition holds.

Proposition 4.8. *If there is no path in G'_k from o_k to d_k passing through arc $a \in A(G'_k)$, then*

$$f_a^{sk} = 0 \qquad \forall s \in S. \tag{23}$$

Table 1: Comparison between formulations RSA-BASE and RSA-EXT for objective function O_1.

Instance		RSA-BASE				RSA-EXT			
Net.	$\|K\|$	CPU	LB	gap	nb	CPU	LB	gap	nb
Spain	15	2.7	18	0	0	0.2	18	0	0
Spain	20	6.3	24	0	0	0.3	24	0	0
Spain	25	38.6	37	0	21	0.9	37	0	0
NSF	30	4167	69	0	165	16.5	69	0	0
NSF	40	2468	90	0	0	38.0	90	0	0
NSF	50	7200	98	-	-	37.9	98	0	0
Germ.	40	7200	94	-	-	51.6	95	0	0
Germ.	50	5194	82	0	0	43.3	82	0	0
Germ.	60	7200	176	-	-	77.4	180	0	0

Table 2: Comparison between formulations RSA-BASE and RSA-EXT for objective function O_2.

Instance		RSA-BASE				RSA-EXT			
Net.	$\|K\|$	CPU	LB	gap	nb	CPU	LB	gap	nb
Spain	15	0.9	5680	0	0	0.2	5680	0	0
Spain	20	3.5	8150	0	0	0.3	8150	0	0
Spain	25	108	10830	0	226	0.8	10830	0	0
NSF	30	252	24018	0	0	15.8	24018	0	0
NSF	40	315	33253	0	0	36.5	33253	0	0
NSF	50	1856	34431	0	0	43.6	34431	0	0
Germ.	40	7200	12615	-	-	52.5	12615	0	0
Germ.	50	5039	9125	0	0	41.6	9125	0	0
Germ.	60	7200	14184	-	-	55.9	28516	0	0

For identifying the variables that can be eliminated as a consequence of Proposition 4.8, it suffices to construct graphs G'_k and run a preprocessing similar to length preprocessing.

5 COMPUTATIONAL RESULTS

In order to confirm the relevance of our approach, in this section we evaluate the computational performances achieved with the proposed extended compact formulation. For this, three network topologies are investigated: Spain, NSF and German [2, 15]. Spain topology has 5 nodes, 7 edges and 30 slots. NSF topology has 9 nodes, 13 edges and 120 slots. German topology has 17 nodes, 25 edges and 140 slots. For each network topology three sets of randomly generated demands are evaluated. Each considered demand requires either 3, 5 or 6 slots and supports, respectively, 100 Gb/s (3000 km reach), 200 Gb/s (1500 km reach), or 400 Gb/s (600 km reach).

Tables 1-4 provide a sample of the performances obtained with each formulation using each of the objective functions described in Section 2. All the experiments were performed using the state-of-the-art MIP solver CPLEX 12.10 on a computer equipped with a 1.60 GHz Intel Core i5-8265U processor and 16 Gb RAM. A time limit of two hours was imposed in each run. For each formulation, the total time in seconds required for the optimization is displayed under column CPU. The best lower bound obtained is provided under column LB. If the time limit of two hours is exceeded, the remaining gap percentage is displayed under column gap. The best solution cost (UB) obtained is not displayed but can be deduced since gap $= \frac{\text{UB}-\text{LB}}{\text{UB}}$. The absence of results for some of the instances indicates that the model breaks down and no feasible integer solution was found within the two hours time limit. Finally, the number of nodes explored in the enumeration tree is given under column nb.

On all tested instances, we could either prove optimality faster or the remaining gap by the end of the time limit was smaller

Table 3: Comparison between formulations RSA-BASE and RSA-EXT for objective function O_3.

Instance		RSA-BASE				RSA-EXT			
Net.	$\|K\|$	CPU	LB	gap	nb	CPU	LB	gap	nb
Spain	15	2510	15	0	29136	4.2	15	0	5394
Spain	20	7200	15	31.8	24767	7.0	22	0	5279
Spain	25	2399	29	0	4527	2.2	29	0	0
NSF	30	7200	15	-	-	7200	46.4	3.3	4615
NSF	40	7200	10	-	-	7200	30.3	42.9	0
NSF	50	7200	21	-	-	7200	34.2	71.5	0
Germ.	40	7200	6	-	-	7200	17.2	87.7	0
Germ.	50	7200	8	-	-	7200	20.7	85.2	0
Germ.	60	7200	7	-	-	7200	31	-	-

Table 4: Comparison between formulations RSA-BASE and RSA-EXT for objective function O_4.

Instance		RSA-BASE				RSA-EXT			
Net.	$\|K\|$	CPU	LB	gap	nb	CPU	LB	gap	nb
Spain	15	218	117	0	6206	0.2	117	0	0
Spain	20	5370	217	0	56471	0.4	217	0	0
Spain	25	3755	341	0	6775	0.5	341	0	0
NSF	30	7200	508	-	-	273	598	0	6688
NSF	40	7200	540	-	-	778	788	0	6278
NSF	50	7200	716	-	-	3249	1053	0	7333
Germ.	40	7200	270	-	-	889	461	0	7063
Germ.	50	7200	366	-	-	7200	593	1.3	2478
Germ.	60	7200	371	-	-	7200	1308	12.7	0

using formulation RSA-EXT than when using RSA-BASE. For objectives O_1 and O_2, the extended formulation could solve all tested instances to optimality within less than 1.5 minute, while the original formulation struggled substantially more. Indeed, 5 instances could not be solved to optimality within the time limit of 2 hours. Considering the instances that could be solved by both formulations, RSA-EXT was in average 65.7 times faster than RSA-BASE. For objectives O_3 and O_4, RSA-BASE could only solve the instances tested over the Spain network, and for all other topologies it could not even find a feasible solution. RSA-EXT, in return, only failed to find a feasible solution for the largest instance tested (*i.e.*, German, 60 demands) with objective O_3. Considering the instances that could be solved by both formulations, RSA-EXT was in average 4742 times faster than RSA-BASE. Such computational results strongly confirm the efficiency of the proposed extended formulation.

6 CONCLUDING REMARKS

Objective functions related to efficient spectrum assignments were found to be much harder to optimize than the objective functions related to routing aspects. This has probably to do with the fact that when trying to fit all demands within a smaller subspectrum width, the non-overlapping requirements become increasingly restrictive and hence the interactions between demands grow to be progressively conflicting. Moreover, such spectrum related objectives induce a great degree of symmetry in the MILP. Symmetry-breaking techniques might be useful to be considered in these cases. As the number of demands to be routed increases, the root node becomes progressively harder to be solved. A promising idea is to consider other methods (*e.g.*, Lagrangian relaxation) for obtaining good approximations of the linear relaxation values.

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