Optimal location of loading/unloading bays in urban areas, model and case study

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1 INTRODUCTION

We address the problem of determining the optimal number and location of the so called loading/unloading bays (L/U bays), i.e., slots within the urban parking space, reserved to perform operations of delivery and/or pick-up of goods to/from a set of given clients. We extend an existing mixed integer-linear programming model for the location of slots and demand assignment, by adding constraints which allow to control both the distance between clients and slots, and the splitting of required parking time for each client. We apply and compare both models using a case study related to the central business district of a medium-sized city.

KEYWORDS

city logistics, loading/unloading bays, optimal location and assignment, mixed integer linear programming

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[11], a two-step methodology for the L/U bays location problem is proposed. First, the minimum number of loading/unloading zones required to fulfill the demand of a given set of commercial establishments is determined (quantification problem). Then, a mathematical programming model is suggested to determine the optimal location for the L/U bays (location-allocation problem). Two different alternatives are considered for the objective function of the model: one consists in allocating the bays near to the most demanding establishments, and the other one minimizes the maximum product of distance and demand for each one of the establishments. Also, in [12] a two-step methodology for the L/U location and size problems is considered. In the first stage a mathematical programming model is proposed for the problem of location and sizing of the lay-by areas. The second stage addresses the performance assessment and tuning of the size of the lay-by areas by means of a simulation model. In [7], the problem of determining the most relevant delivering or picking up parcels of urban freight vehicles is studied by means of a cluster analysis based on GPS traces and a vehicle observation survey. In general terms, we note that the dynamic nature of urban freight operations has motivated the development of simulation models which represent the system with a high level of detail. These simulation models have been embedded into optimization models which are solved heuristically in order to achieve near-optimal solutions to the problem of location of L/U bays. On the other hand, another stream of literature propose optimization models which include sub-models to represent the freight operations over a set of L/U bays. These sub-models are typically static assignment models, which constitute an approximation of the dynamic simulation ones, using mean values of the problem parameters (mostly parking time) instead of their distributions of probability. The advantage of this approach lies in the possibility of computing optimal solutions regarding the location of L/U bays.

In this work, we propose an extension to the optimization model introduced in [11] and apply it to a real case based on one of the main commercial districts of Montevideo (Uruguay), known as Ciudad Vieja, the historical downtown of the city. The original model of [11] aimed to select an optimal subset of bays from a given set of candidates, minimizing the total distance between clients and bays, and fulfilling the demand requirements of the clients expressed as parking times. Based on practical concerns about operative restrictions in the logistic sector, we add new constraints in order to enrich the original model: the first one states that any client must fulfill its demand from bays located no longer than a given distance, while the second one ensures that the demand (requested parking time) can not be split into fractions smaller than a given parameter. The model is applied to the real case, to investigate the effect of the new constraints over the solutions.

2 MODEL

The following model is aimed to support strategic decisions related to the number and location of L/U bays. Its main goal is to ensure the best possible solution from the point of view of the logistic operators, subject to a restriction on the usage of public space. An implicit assignment of clients to bays is modeled in order to evaluate the solution according to the goal of the model.

Let \( I \) be the set of \( n \) potential places to locate bays and \( J \) the set of \( m \) clients. Moreover, let \( C_i \) be the capacity of bay \( i \), \( D_j \) the demand of client \( j \) (both in time units), \( d_{ij} \) the distance between bay \( i \) and place \( j \) and \( N \leq n \) the maximum number of bays to be open. Decision variables include \( y_i \) (open or not bay \( i \)), \( z_{ij} \) (discrete assignment of client \( j \) to bay \( i \)) and \( x_{ij} \) (demand assignment). The formulation is as follows:

\[
\begin{align*}
\min & \quad \sum_{i=1}^{n} \sum_{j=1}^{m} d_{ij} x_{ij} \\
\text{s.t.} & \quad \sum_{i=1}^{n} y_i \leq N, \\
& \quad \sum_{j=1}^{m} x_{ij} = D_j, \quad \forall j \in J, \\
& \quad \sum_{i=1}^{m} x_{ij} \leq C_i y_i, \quad \forall i \in I, \\
& \quad T_{\text{min}} y_i \leq \sum_{j=1}^{m} x_{ij}, \quad \forall i \in I, \\
& \quad x_{ij} \leq C_i z_{ij}, \quad \forall i \in I, \quad \forall j \in J, \\
& \quad x_{ij} \geq T_{\text{min}} z_{ij}, \quad \forall i \in I, \quad \forall j \in J, \\
& \quad d_{ij} z_{ij} \leq D_{\text{max}}, \quad \forall i \in I, \quad \forall j \in J, \\
& \quad x_{ij} \geq 0, z_{ij} \in \{0,1\}, \quad \forall i \in I, \quad \forall j \in J, \\
& \quad y_i \in \{0,1\}, \quad \forall i \in I.
\end{align*}
\]

Expressions (1)-(4) represent a classical location and assignment problem and constitute the original base model proposed in [11]; constraint (5) prevents from opening unused bays. By means of the new discrete variable \( z \) and parameters \( T_{\text{min}} \) (minimum value for splitting demand) and \( D_{\text{max}} \) (maximum distance for assigning clients to bays) we are able to extend the original model with the new constraints (6)-(8). Since we add variable \( z \) to represent the assignment, the problem may turn computationally more difficult to solve.

Note that the only decision variables which are under direct control of the decision maker are \( y \). The model also performs an assignment of clients to bays in order to compute the performance of the system, once decisions \( y \) are made. This is represented by variables \( x \) and \( z \). While the latter correspond to discrete assignments (note that one client may be assigned to different bays in order to fulfill its demand), the former correspond to the demand assignment of each client to the different bays (note that the demand, which is an amount of time, can be split among different bays). Therefore, although this assignment may reasonably represent the real behavior of logistics operators under general conditions, the optimal solution obtained from the model may not be very realistic, for example, if the assignment is divided into two bays, one of them to satisfy a very small portion of the demand. That is the reason why we consider the new constraints, which are discussed in the following:

- Even though assigning a client to a faraway bay may be efficient from the point of view of the whole solution, it would result in an unrealistic assumption. When translated to the real system, logistic operators assigned to a bay which is far from the client will prefer to double-park instead, which is one of the negative consequences that we want to avoid (or at least, to minimize) by means of a systematic planning of the bays’ locations. To this end, we add constraint (8) of maximum distance, which restricts the alternatives for assignment.
- The demand of each client is expressed as an amount of time within the planning horizon. It is an aggregated value,
which means that we do not know how many operations of loading/unloading (nor their durations) will be performed in relation to each client. This is a reasonable assumption, since feeding the model with highly disaggregated data may lead to results that are not stable in time. Moreover, collecting disaggregated data on urban freight operations may be a challenging and costly task [3]. On the other hand, considering different planning horizon lengths and times (i.e., different time windows), would allow us to evaluate the model under variations in the demand values as in [8]. However, the original model of [11] allows for potential splitting of the demand into arbitrarily (and not realistic) low values. To avoid these non-desirable assignments, we add constraint (7) of minimum amount of time for demand splitting. Unlike constraint (8), where parameter \(D_{\text{max}}\) can be configured by common sense and universal criteria, the configuration of parameter \(T_{\text{min}}\) requires stating assumptions about the nature of the demand of each context. Thus, for scenarios where the demand is composed by short operations, the value of \(T_{\text{min}}\) may be small as well, which will be convenient from the point of view of the whole optimization model since it represents the less restrictive condition. On the contrary, if we do not have information about the composition of the demand of clients, a larger value of \(T_{\text{min}}\) will be more conservative and therefore, potentially more compatible with the real situation. Finally, we note that despite the random nature of the demand for delivery times, as shown in [14] and [9], we decided to keep the model as simple as possible in favor of its usability from the optimization point of view. Regarding the mathematical structure of problem (1)-(10), we may classify it as a variant of the Capacitated Facility Location Problem (CFLP) [16], which has been formulated in the literature using Mixed Integer Linear Programming (MILP). In our variant, the objective function considers only variable costs (distance multiplied by demand). While fixed costs incurred by opening bays are not part of the objective function, we consider a maximum number of bays to be open through constraint (2). Moreover, capacities are imposed by constraint (4), stating that the demand that can be collectively assigned to any bay (potentially from several different clients) should not exceed the extent of the planning horizon. Several solving methods (both exact and heuristics) have been proposed to solve the CFLP, since it is computationally hard to solve [4]. In this work, we investigate empirically the computational tractability of our problem, by applying a state-of-the-art MILP solver to the instance corresponding to our case study. The model was coded in AMPL and solved with CPLEX 20.1.0.0 on a PC with 16 CPUs Intel Core i9-9900K 3.60GHz, 64 bit, 64GB RAM, and CentOS Linux 7.

3 RESULTS AND DISCUSSION

The model was applied to a case study in Ciudad Vieja, the historical downtown of Montevideo, main city of Uruguay. The area concentrates many administrative activities, including both public and private offices, as well as several banks. Tourism is intense also during some parts of the year. Several shops serve all these activities, including office suppliers and restaurants. This is a highly dense urban center with high levels of traffic congestion, which causes that activities associated with loading and unloading are often complex. The area is constituted by an almost perfect grid of 13 × 8 blocks of 100 meters each. Most of the streets are narrow and few of them are of pedestrian type.

In order to build the case study, potential locations of L/U bays were determined by visual inspection using Google Earth. As a criterion, a potential bay was considered as every segment of 8 meters of clear sidewalk. This length was selected as it is able to accommodate the largest vehicle allowed in the area by the local freight regulation. Location of the clients were determined by a cadastral survey of the zone. This survey, performed by the municipality (Intendencia de Montevideo, IM), classifies establishments according to ISIC 1. The demanded parking time was determined based on each ISIC establishment using literature estimates [5], resulting values \(D_j\) in the range \([37, 1825]\) (minutes). The capacity for each bay, \(C_i\), is set to 1440 minutes (the whole day). Manhattan distances between bays and clients were computed to configure parameter \(d_{ij}\). As a reference value, the number of open bays currently in the area is 36 (taken from the online Geographical Information System of IM2).

After processing the data as described above, the number of potential places where bays can be located is \(n = 286\), and the number of clients is \(m = 290\). Given the size of the whole study area, clearly is not suitable to open one bay for each client. Under this assumption, we ran the original model of [11] (setting \(D_{\text{max}}\) to an arbitrary large value equal to 9999 and \(T_{\text{min}} = 0\)) to investigate its response for several values of the maximum number of bays to be open, \(N\). The main finding is that the minimum value of \(N\) for which a feasible solution is found (w.r.t. constraint of maximum capacity (4)) is 38, while for \(N \geq 144\) the best possible objective value is attained. The execution time ranges from less than one second to 79 seconds. Figure 1 plots the percentage gap of the objective value for different values of \(N\) within the range \([39, 144]\), with respect to the best one, corresponding to \(N = 144\). This simple sensitivity analysis shows that the quality of the solution (in terms of objective value) can be significantly improved (e.g. moving the gap from 60% to 30%) by opening 52 bays instead of 39. The improvement is monotonic w.r.t. \(N\) as we might expect, however, it should be taken into account that as we increase \(N\), we are reducing the availability of public space which may be needed for other purposes.

![Figure 1: Sensitivity analysis with respect to the maximum number of bays to be open.](https://unstats.un.org/unsd/classifications/Econ/ISIC.cshtml)

In order to study the effect of adding constraints of maximum distance and minimum parking time, a sensitivity analysis

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1https://unstats.un.org/unsd/classifications/Econ/ISIC.cshtml
2https://sig.montevideo.gub.uy/
was performed. We consider the parameter \(D_{\text{max}}\) within the set \([200, 300, 400, 500, 600]\) in meters, and parameter \(T_{\text{min}}\) within the set \([10, 15, 20, 25, 30]\) in minutes; both distance and time values were selected by considering reasonable conditions in the context of the case study. For each value of \(N\) taken from the set \([40, 41, 45, 60, 100]\), the model was executed for the 25 combinations of \(D_{\text{max}}\) and \(T_{\text{min}}\) resulting from crossing their respective sets of possible values. The execution time of the restricted model did not change significantly with respect to that of the unrestricted one. In Table 1 we summarize for each \(N\) from the set under consideration, the percentage gap (in terms of objective value) of each restricted solution with respect to the unrestricted one corresponding to \(D_{\text{max}} = 9999\) and \(T_{\text{min}} = 0\), i.e., the one of the original model of [11]. Both minimum and maximum gap values are shown, as well as averages computed from the solutions corresponding to the 25 combinations of values of \(D_{\text{max}}\) and \(T_{\text{min}}\). We can observe that as \(N\) decreases from 100 (a rather unrealistic value, considering the number of bays open in the current scenario) to 40, the average gap increases in several orders of magnitude. The maximum gap corresponds to \(N = 40, D_{\text{max}} = 200\) and \(T_{\text{min}} = 30\), i.e., the more restricted scenario (where each client must have a bay no further than 200 meters and the demand can not be split into fractions smaller than 30 minutes). For \(N = 39\), no feasible solution was obtained for \(D_{\text{max}} = 200\). With this experiment, we show that by adding constraints which are meant to bring more realism to the original model of [11], we obtain solutions which are not so different to the ones corresponding to the original model in terms of objective values. In the following analysis, we investigate the differences in terms of the values of the decision variables.

In order to have more insight on the differences between solutions of both unrestricted and restricted models, we compare the values of decision variables of the optimal solution for \(N = 40, D_{\text{max}} = 9999, T_{\text{min}} = 0\) (unrestricted solution) and for \(N = 40, D_{\text{max}} = 200, T_{\text{min}} = 30\) (the more restricted one).

First, we compare decisions related to open or not each potential bay, i.e., decision variable \(y\). The first observation is that both solutions decided not opening the same set of 237 bays and decided opening the same set of 31 bays. Moreover, the unrestricted solution and the restricted one select different sets of bays to complete the remaining 9 bays. This means that both solutions are identical in 78% w.r.t. decisions made about bays to be open. To investigate the practical consequences in the assignment of the difference between solutions, we analyze the values of assignment (decision) variables \(z\) and \(x\). In Fig. 2 we plot a histogram of the distances of assignments of clients to bays for solutions of both unrestricted and restricted models. As we might expect, the distances of the restricted model do not surpass 200 meters, since we imposed that value for parameter \(D_{\text{max}}\). Moreover, although the assignments are similar in general terms, the unrestricted model exhibits more assignments in the range of large distances, which represents an advantage for the restricted model.

![Figure 2: Histogram of assigned distances of clients to bays.](image)

The same analysis was made with the assigned demand, i.e., time requested for each client, which can be split among different bays. From Fig. 3 we can observe that, as we might expect, the restricted model does not assign fractions of demand smaller than 30 minutes, since we imposed that value for parameter \(T_{\text{min}}\). We do not identify any tendency in the assignment. A closer inspection reveals that out of 290 clients, 27 of them split their demands into 2 fractions and 3 of them split their demands into 3 fractions in the unrestricted model. Regarding the restricted model, we observe that 29 clients split their demands into 2 fractions and only one of them splits its demand into 3 fractions. Figure 4 shows the optimal solutions for both unrestricted and restricted models. Purple points represent clients. Red, orange and green points represent opened bays with available capacity in the ranges \([0\%\%, 20\%\%]\), \([20\%\%, 50\%\%]\) and \([50\%\%, 100\%]\), respectively. Grey points represent unopened bays. The clients’ assignments to bays are represented by light blue lines.

![Figure 3: Histogram of demand assigned to bays.](image)

In summary, by applying the original model of [11] to our case study, we were able to find the lowest feasible number of L/U bays

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**Table 1: Summary of gaps between optimal values of the original model and the extended one**

<table>
<thead>
<tr>
<th>(N)</th>
<th>min. gap (%)</th>
<th>max. gap (%)</th>
<th>avg. gap (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>0.003</td>
<td>3.625</td>
<td>0.735</td>
</tr>
<tr>
<td>41</td>
<td>0.004</td>
<td>2.382</td>
<td>0.486</td>
</tr>
<tr>
<td>45</td>
<td>0.001</td>
<td>0.626</td>
<td>0.131</td>
</tr>
<tr>
<td>60</td>
<td>0.008</td>
<td>0.038</td>
<td>0.025</td>
</tr>
<tr>
<td>100</td>
<td>0</td>
<td>0.005</td>
<td>0.003</td>
</tr>
</tbody>
</table>
to be open, which is very similar to the current number of open bays in the area. Then, by extending the model with additional distance and time constraints, we show that the objective value does not change noticeably, i.e., the quality of the solution is not affected. However, we obtained assignments of clients to bays which are more likely to be consistent with the behavior of the real system.

Finally, with the aim of comparing the current solution implemented in the real system against the optimal solution delivered by our proposed model, we fix variables \( y \) according to the bays currently open (totaling 36) and we let the model to assign clients to bays, in order to compute the objective value. Then, we impose a maximum number of bays equal to 36 and we let the model choose the best set of bays to be open, along with its corresponding assignment of clients. It is worth noting that we had to reduce the demand of all clients down to 90%, since for less than 39 bays the problem has not feasible solution. Also, we relaxed the constraint of maximum distance, since in the current solution there are bays that are located beyond the distance considered in our model. The results show that the optimized solution improves slightly in comparison with the ones derived from the original model. These differences do not imply major changes in the objective value. However, differences in the geographical location of the bays of the optimal solutions are observed, as well as in the assignment of clients to bays and the splitting of demand among bays. Finally, we consider that the most relevant aspect of the solutions of the extended model is that they capture in a more realistic way the behavior of the logistic operators. This aspect could be considered more precisely in future studies by explicitly adding the point of view of the logistics operator using a bi-level model formulation. In this way, we could decouple decisions of the different participants involved in the problem, which, at the same time, are coupled into a single optimization problem aimed to be solved by a central planner.

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REFERENCES


